

# On the Prisoner's Dilemma in R&D with Input Spillovers and Incentives for R&D Cooperation

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## Abstract

This paper considers a standard model of strategic R&D with spillovers in R&D inputs, and extends the result that duopoly firms engaged in a standard two-stage game of R&D and Cournot competition end up in a prisoner's dilemma situation for their R&D decisions, whenever spillover effects and R&D costs are relatively low. In terms of social welfare, this prisoner's dilemma always works to the advantage of both consumers and society. This result allows a novel and enlightening perspective on some issues of substantial interest in the innovation literature. In particular, the incentive firms face towards R&D cooperation in the form of an R&D cartel is shown to be maximal for the case of zero spillovers, which is when the prisoner's dilemma has the largest scope.

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# 1 Introduction

Technological innovation is broadly viewed as the engine behind economic growth. This is clearly reflected in the modern theory of endogenous growth (Romer, 1986). Innovating firms and entrepreneurs are typically described in very positive terms, often bordering on heroic, in popularized writings in economics. Yet, a recent small strand of the literature on innovation in industrial organization has uncovered the simple but insightful result that innovating firms in imperfectly competitive markets often undertake innovative activities as a consequence of being caught in a simple prisoner's dilemma situation. In other words, in some circumstances, firms only respond to powerful strategic forces when they engage in process research and development (or R&D), in the sense that this decision forms a dominant strategy, even though remaining under the status quo ante (with no R&D) would actually be to their mutual benefit. Thus, firms might sometimes conduct R&D with the sole intent of keeping up with their rivals, in what may be viewed as a destructive race from their joint point of view.

Bacchiega et al. (2010) considers a modified version of one of the commonly used models of strategic R&D, due to d'Aspremont and Jacquemin (1988), henceforth AJ. This model postulates a standard symmetric two-period framework of duopolistic process R&D/Cournot competition with spillover effects that are additive in cost reductions (or R&D outputs). Bacchiega et al (2010) add an initial period to this common model, wherein firms decide simultaneously whether or not to conduct R&D. In the resulting three-stage game, they find that firms are caught in a prisoner's dilemma in R&D whenever spillover effects are sufficiently small. Amir et al. (2011) consider a similar question and derive a similar result of a prisoner's dilemma in R&D but do so within the context of the original two-stage game of AJ. To be able to even address this question, the latter study proposed a definition of a prisoner's dilemma for games where the action set (for the R&D decision) is not necessarily binary but is instead uncountable.<sup>1</sup>

The present paper addresses the same question, i.e., investigating the possibility of a prisoner's dilemma in R&D and characterizing its scope, but does so in the context of the other competing model of duopolistic process R&D/product market competition, which postulates spillover effects

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<sup>1</sup>See also Amir, Halmenschlager, and Jin (2011a) for another related context where a prisoner's dilemma in R&D shows up.

that are additive in monetary expenditures on R&D (or R&D inputs). This is a much older model, initially proposed by Ruff (1969) and subsequently considered by Spence (1984), Kamien, Muller and Zang (1992), henceforth KMZ, and Amir, Evstigneev and Wooders (2003). Amir (2000) provides an extensive comparison of these two competing models, and finds that they differ in some important ways, both in terms of some basic theoretical predictions and sometimes also in terms of policy prescriptions. In the way of final conclusion, Amir (2000) suggests that the KMZ model is more likely to be applicable across the board to generic industries than the AJ model, the latter embodying in a somewhat hidden manner that a firm's R&D and that of its rival's may constitute perfect complements (i.e, without any redundancies) and that adding up cost reductions can circumvent the decreasing returns aspect of process R&D. For a more detailed analysis and consideration of further points of contrast, the reader is referred to the original paper.<sup>2</sup>

In view of the depth of some of the divergences between the two models, any conclusion that holds in one of the models could not be considered as necessarily holding in the other. Therefore, it makes sense to investigate the presence and extent of the prisoner's dilemma in the KMZ model too. Furthermore, since this is now considered as being the more central model, the question at hand acquires some further interest. Finally, this paper may in any case be viewed as a robustness check of the existing papers on this topic.

The main result of this paper turns out to be qualitatively the same as in the recent literature using the AJ model, namely that a prisoner's dilemma in R&D does hold for the KMZ model, and has a scope that is qualitatively similar to the one found for the other model. The prisoner's dilemma in R&D tends to hold for sufficiently small values of the spillover parameter and of the cost of R&D (the latter captured by a simple parameter in both models). In particular, it always holds in a neighborhood of zero spillovers, regardless of the cost of R&D.

The fact that a prisoner's dilemma in R&D holds when the spillover parameter and the cost of R&D tend to be small is intuitively plausible. Indeed, a higher cost of R&D always leads in a natural way to a lower propensity on the part of firms to engage in R&D, and thus indirectly mitigates the bite of the prisoner's dilemma effect. Similarly, high spillovers are always expected to

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<sup>2</sup>Other studies that elaborate on some aspects of the differences between these two models include the following: Hinloopen and Vandekerckhove (2009, 2011), Martin (2002), Hauenschild, (2003), Tesoriere (2008), Jin and Troege (2006), Stepanova and Tesoriere (2010), and Piga and Poyago-Theotoky (2005).

dilute firms' incentives for R&D through the well known free rider effect of the commons, and thus also tend to attenuate the effects of the prisoner's dilemma.

While R&D competition will tend to ruin the firms involved in it, it fortunately works to the benefit of consumers and society as a whole, just as one would expect. A (second-best) welfare analysis confirms this well known and rather intuitive point: The prisoner's dilemma at hand is always beneficial to consumers. In addition, this benefit is so large that it overcomes the negative effect on firms' profits and always leads to an increase in social welfare.

Imperfect appropriability, also often referred to as knowledge externalities or spillovers, is a central feature of technological innovation. Uncovering the presence of, and the specific conditions for, a prisoner's dilemma in R&D provides an interesting perspective on several aspects of firms' strategic innovative behavior, including on the effects of imperfect appropriability of R&D. First, observe that under low spillovers, firms' R&D behavior is not affected by the public goods externality in a significant way, but reflects instead the familiar grip of a prisoner's dilemma, while at high spillovers, the presence of spillovers alone is detrimental to firms' innovative efforts and equilibrium profits. The latter point is widely accepted and documented in the literature on innovation (see e.g., Griliches, 1995 or Spence, 1984). Thus, substantial imperfect appropriability and the R&D prisoner's dilemma may be viewed as (mutually exclusive) substitutes in pulling firms' fortunes down. As a consequence, one might well expect that mid-level spillovers would be preferred by firms to either low or high spillovers. This is indeed the case, as shown below (see De Bondt et al., 1992 for the analogous result in the context of the AJ model).

In addition, the prisoner's dilemma result can also be invoked to provide a clarifying perspective on the incentive firms face for forming R&D cartels, as an alternative to non-cooperative R&D. Taking the (positive) difference between cartel and non-cooperative profits as a natural quantitative measure of this incentive, we show that it is U-shaped in the spillover rate. More precisely, this incentive is very high (in fact maximal) when the spillover rate is equal to 0, also very high when the spillover rate is close to 1, but very weak for mid-level spillovers (and non-existent when the spillover rate is exactly 50 per cent). Under high spillovers, forming an R&D cartel is a classical way of internalizing the large knowledge externality, as is widely recognized (see, e.g., Bernstein and Nadiri, 1988). On the other hand, under low spillovers, going for an R&D cartel is something

that standard intuition from basic models of strategic R&D does not readily explain. This applies particularly to the fact that the incentive to form an R&D cartel is maximal at zero spillovers. However, in light of the prisoner’s dilemma, whose grip we show is maximal at zero spillovers, one easily appreciates the fact that firms would regard an R&D cartel as a very natural way out of the prisoner’s dilemma grip.

In view of the aforementioned welfare results, by inducing firms to engage in non-cooperative R&D, the prisoner’s dilemma at hand ends up working in favor of both consumers and society. This suggests that antitrust authorities should perhaps revise their permissive attitude towards R&D cartels, and consider a more discriminating policy towards R&D cartels, one that would be negatively disposed towards R&D cooperation in industries with low spillovers.

This paper is organized as follows. Section 2 summarizes the basic KMZ model and its main results of interest to the present paper. Section 3 provides a description of the three-stage game and a characterization of the parameter region under which a prisoner’s dilemma holds. In addition, this section also provides some new insight into firms’ equilibrium profits in light of the prisoner’s dilemma along with the associated welfare analysis. Section 4 compares the present result to the analogous result for the AJ model. Section 5 discusses some of the possible implications of the main result of this paper for the well known comparison between R&D cooperation and non-cooperation.

## 2 The KMZ model: Spillovers in R&D inputs

We begin by reviewing the well-known model by Kamien et al. (1992), or KMZ, since this is a central building block of our three stage game. The KMZ model is based on the standard two-stage game of R&D/product market competition as follows. In stage 1, firms decide on R&D expenditures, and on stage two, firms engage in Cournot competition in a market for a homogeneous good with inverse demand  $P(q_1 + q_2) = a - b(q_1 + q_2)$ , where  $b > 0$  and  $q_1$  and  $q_2$  are the outputs of firms 1 and 2. Both firms have common *initial* marginal cost  $c$ , with  $0 < c < a$ . The R&D process follows the version of the well-known model of KMZ as adapted by Amir (2000). Let  $y_1$  and  $y_2$  be the expenditures on R&D by firms 1 and 2. The resulting cost reduction for firm  $i$  is then  $\sqrt{\frac{1}{\gamma}(y_i + \beta y_{-i})}$  where  $\gamma > 0$  is a measure of the effectiveness of R&D and  $\beta \in [0, 1]$  is a spillover

parameter. Therefore, their post R&D unit cost is then

$$c_i = c - \sqrt{\frac{1}{\gamma}(y_i + \beta y_{-i})}$$

Here,  $\beta$  is a spillover parameter that captures the degree of involuntary leakage of private R&D results from a firm to its market rival. Thus spillovers are postulated to take place in R&D inputs here, in contrast to the other, competing model by d'Aspremont and Jacquemin (1988), which postulates spillovers in R&D outputs, as discussed in detail in Amir (2000). Both specifications clearly reflect the commonly accepted feature of decreasing marginal returns to R&D.

Due to the presence of spillover effects, the action sets for the R&D phase of the game are inter-dependent and given by

$$\Omega_i = \{y_i : y_i \leq c^2\gamma - \beta y_j\}, i, j = 1, 2, i \neq j.$$

In order to guarantee that the post R&D situation will have both firms active in a duopoly market (i.e., avoid an outcome of monopoly with one firm endogenously exiting), we need to make the following standard assumption (maintained throughout the paper).

**(A1)**  $a > 2c$ .

Given **(A1)**, the profit function of firm  $i$ , conditional on the subsequent Cournot equilibrium, as a function of private R&D expenditure levels  $y_1$  and  $y_2$ , is

$$\begin{aligned} \Pi_i(y_1, y_2) &= \frac{1}{9b} \left[ a - 2 \left( c - \sqrt{\frac{1}{\gamma}(y_i + \beta y_j)} \right) + c - \sqrt{\frac{1}{\gamma}(y_j + \beta y_i)} \right]^2 - y_i \\ &= \frac{1}{9b} \left[ a - c + 2\sqrt{\frac{1}{\gamma}(y_i + \beta y_j)} - \sqrt{\frac{1}{\gamma}(y_j + \beta y_i)} \right]^2 - y_i \end{aligned}$$

There is a unique and symmetric equilibrium  $(y^*, y^*)$ , which can be easily computed (see e.g., Amir, 2000)<sup>3</sup>

$$y^* = \begin{cases} \frac{\gamma(2-\beta)^2(a-c)^2}{(1+\beta)(9b\gamma-2+\beta)^2} & \text{if } b\gamma > \frac{(2-\beta)a}{9c} \\ \frac{\gamma c^2}{(1+\beta)} & \text{if } b\gamma \leq \frac{(2-\beta)a}{9c} \end{cases} \quad (1)$$

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<sup>3</sup>This requires a second-order condition which can be shown to be (see e.g., Amir, Evstigneev and Wooders, 2003)

$$9b\gamma > (2-\beta)$$

Since this is most demanding for  $\beta = 0$ , it is sufficient to assume that  $9b\gamma > 2$ .

We shall refer below to the top line in (1) as the interior, and to the bottom line in (1) as the boundary, equilibrium or solution.

The corresponding per-firm equilibrium profit is

$$\Pi^* \equiv \Pi_1(y^*, y^*) = \begin{cases} \frac{\gamma [9b\gamma(1 + \beta) - (2 - \beta)^2] (a - c)^2}{[9b\gamma - (2 - \beta)^2] (1 + \beta)} & \text{if } b\gamma > \frac{(2 - \beta)a}{9c} \\ \frac{a^2(1 + \beta) - 9b\gamma c^2}{9b(1 + \beta)} & \text{if } b\gamma \leq \frac{(2 - \beta)a}{9c} \end{cases} \quad (2)$$

### 3 The three-stage game

In the present paper, we consider a three-stage game between the two firms as follows. In the first stage, the firms commit to an irreversible decision, whether to conduct any R&D or not. In other words, they have a binary choice, where the actions will be referred to as Invest and Not Invest. In the second stage, each firm that chose Invest in the first stage decides on the level of R&D, while a firm that chose Not Invest is committed to spending zero on R&D. In the third stage, firms engage in Cournot competition in a market for a homogeneous good with a linear inverse demand  $P = a - b(q_1 + q_2)$ , where  $q_1$  and  $q_2$  are the outputs of the two firms.

It follows that if both firms choose Invest in the first stage, the subgame that follows is simply the well-known model by KMZ. On the other hand, if both firms choose Not Invest in the first stage, the subgame that follows is simply the standard Cournot oligopoly with no R&D. Finally, if one firm chooses Invest and the other chooses Not Invest, then the ensuing subgame pits an R&D-conducting firm against a no-R&D firm, and the optimal investment level of the former must then be evaluated, along with the corresponding profits to both firms.

To analyze the subgame-perfect equilibria of the three-stage game, we calculate the subgame-perfect equilibria of each of the four (two-stage) subgames described above and then collapse the game back to the first-stage decision, Invest or Not Invest. Clearly, this reduction leads to a simple two-action (matrix) game, whereupon checking for a potential prisoner's dilemma is straightforward.

#### 3.1 Reduction to a one-stage R&D game

Upon replacing each of the four subgames starting after each of the four possible action constellations in the first period by its equilibrium payoff, the  $2 \times 2$  normal form R&D investment game

as seen in the first period can be represented as

		Firm 2	
		Invest	Not Invest
Firm 1	Invest	$\Pi_1(y^*, y^*), \Pi_2(y^*, y^*)$	$\Pi_1(\bar{y}, 0), \Pi_2(\bar{y}, 0)$
	Not Invest	$\Pi_1(0, \bar{y}), \Pi_2(0, \bar{y})$	$\Pi_1(0, 0), \Pi_2(0, 0)$

where

(i)  $\Pi_1(y^*, y^*) = \Pi_2(y^*, y^*)$  is as given in (2).

(ii)  $\Pi_1(0, 0) = \Pi_2(0, 0) = \frac{(a-c)^2}{9b}$  is the (standard) Cournot equilibrium profit with no R&D.

(iii)  $\bar{y}$  is a firm's best response to zero R&D by the rival and  $\Pi_1(0, \bar{y}) = \Pi_2(\bar{y}, 0)$  is the corresponding profit level, calculated next.

To find the profit levels when one firm invests while the other does not, suppose firm 1 decides to invest  $y_1 > 0$  while firm 2 chooses not to invest ( $y_2 = 0$ ). Then firm 1's profit is

$$\Pi_1(y_1, 0) = \max_{y_1} \frac{1}{9b} \left[ a - c + 2\sqrt{\frac{y_1}{\gamma}} - \sqrt{\beta \frac{y_1}{\gamma}} \right]^2 - y_1 \quad (3)$$

The first order condition is

$$\frac{2}{9b} \left[ a - c + 2 \left( \frac{y_1}{\gamma} \right)^{1/2} - \left( \beta \frac{y_1}{\gamma} \right)^{1/2} \right] \left[ \frac{1}{\gamma} \left( \frac{y_1}{\gamma} \right)^{-1/2} - \frac{1}{2} \cdot \frac{\beta}{\gamma} \left( \beta \frac{y_1}{\gamma} \right)^{-1/2} \right] = 1$$

Solving yields the best response to zero investment in R&D as

$$\bar{y}_1 = \frac{\gamma(2 - \sqrt{\beta})^2(a - c)^2}{[9b\gamma - (2 - \sqrt{\beta})^2]^2} \quad (4)$$

substitute back to (3) to find firm 1's profit

$$\Pi_1(\bar{y}, 0) = \frac{\gamma(a - c)^2}{9b\gamma - (2 - \sqrt{\beta})^2} \quad (5)$$

If firm 2 chooses zero R&D against  $y_1$  by firm 1, its profit is affected by  $y_1$  via the spillover effect:

$$\Pi_2(y_1, 0) = \frac{1}{9b} \left[ a - c + 2\sqrt{\beta y_1/\gamma} - \sqrt{y_1/\gamma} \right]^2$$



Given firm 1's investment level  $y_1 = \bar{y}$ , firm 2's profit reduces to

$$\Pi_2(\bar{y}, 0) = \frac{(a-c)^2}{b} \left[ \frac{3b\gamma + (2-\sqrt{\beta})(\sqrt{\beta}-1)}{9b\gamma - (2-\sqrt{\beta})^2} \right]^2 \quad (6)$$

### 3.2 Characterizing the prisoner's dilemma

We will say that the three-stage game under consideration embodies a prisoner's dilemma in R&D whenever the two-action game given above constitutes a prisoner's dilemma in the usual sense for the first-stage R&D decisions only (given equilibrium behavior from stage 2 onward).

For such a prisoner's dilemma to hold requires, on the one hand, that investing in R&D be a dominant strategy for both firms, which entails that (say for firm 1)

$$\Pi_1(\bar{y}, 0) > \Pi_1(0, 0) \quad (7)$$

and

$$\Pi_1(y^*, y^*) > \Pi_1(0, \bar{y}), \quad (8)$$

Clearly, (7) always holds since  $\bar{y}$  is the best response to 0. A lengthy but simple computation confirms that (8) holds for the both the interior and the boundary solutions.

On the other hand, such a prisoner's dilemma requires that  $(y^*, y^*)$  be Pareto dominated by  $(0, 0)$ , or  $\Pi_i(y^*, y^*) < \Pi_i(0, 0)$ , which holds (upon simple calculations) for the interior solution if and only if

$$b\gamma < \frac{(1+\beta)(2-\beta)}{27\beta}$$

and for the boundary solution if and only if

$$b\gamma > \frac{(1+\beta)(2a-c)}{9c} \quad (9)$$

We have thus established the following result.

**Proposition 1** *The three-stage game under consideration embodies a prisoner's dilemma in R&D whenever either of the following conditions holds:*

$$\frac{a(2-\beta)}{c9} < b\gamma < \frac{(1+\beta)(2-\beta)}{27\beta} \quad (10)$$

or

$$\frac{(1 + \beta)(2a - c)}{9c} < b\gamma < \frac{(2 - \beta)a}{9c} \quad (11)$$

Here, the meaning is that a prisoner's dilemma holds for the interior solution when (10) holds and for the boundary solution when (11) holds. Accordingly, the overall parameter region where a prisoner's dilemma in R&D holds is depicted in Figure 1 in  $(\beta, \gamma)$  space.

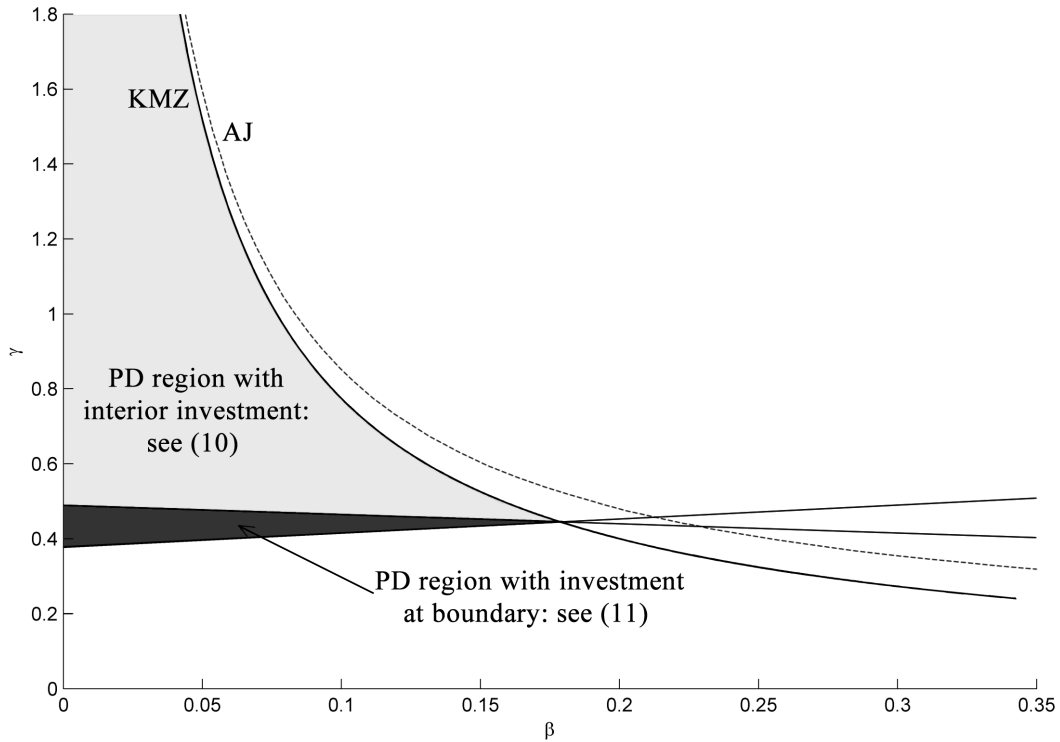


Figure 1: Prisoner's dilemma (PD) region in  $(\beta, \gamma)$  space for the KMZ (solid line) and the AJ (dashed line) model. ( $a = 2.2$ ,  $c = 1$  and  $b = 1$ )

To allow for a direct comparison with the AJ model, Figure 1 also depicts the upper bound for the prisoner's dilemma range in  $(\beta, \gamma)$  space for the AJ model. The two parameter regions where the prisoner's dilemma holds for the two models are quite similar, both qualitatively and quantitatively. On the positive side, this similarity is a welcome result, since it provides a favorable robustness check on the validity of the prisoner's dilemma in R&D (see below for more on this).

Two observations are worthwhile to point out about the range of spillover values for which the prisoner’s dilemma might hold. The first is that it always holds for the important extreme case of zero spillovers, as is easily seen from (10), and by continuity also for sufficiently small spillover values ( $\beta$  close enough to 0).<sup>4</sup>

The second observation is that depending on the particular parameter values one selects (within the scope of the assumptions made in the overall analysis of this model), it can be easily shown that a prisoner’s dilemma can hold only when  $\beta < 0.20$ .<sup>5</sup> Again, a similar conclusion holds for the AJ model (see Bacchiega et al., 2010 and Amir et al., 2011).

While a maximal range of  $[0, 0.20]$  is a relatively small part of the theoretically allowed range of  $[0, 1]$ , it is plausible that this range would actually cover most relevant estimates for actual spillover rates in real-life industries. Naturally, this is an assertion that sounds plausible but clearly awaits empirical justification.

In terms of intuitive perception, the fact that a prisoner’s dilemma in R&D holds when the spillover parameter and the cost of R&D tend to be small is to be expected. Indeed, a higher cost of R&D always leads in a natural way to a lower propensity on the part of firms to engage in R&D, and thus indirectly mitigates the bite of the prisoner’s dilemma effect. Similarly, high spillovers are always expected to dilute firms’ incentives for R&D through the well known free rider effect in public goods environments, and thus also tend to attenuate the effects of the prisoner’s dilemma.

### 3.3 The prisoner’s dilemma and equilibrium profit

This subsection provides further exploration of the interplay between spillovers and the prisoner’s dilemma in R&D, in particular with regard to how this interplay affects firms’ equilibrium profit.

As seen above, with zero spillovers, firms are always caught in a prisoner’s dilemma in R&D (for

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<sup>4</sup>Recall that the AJ and KMZ models are perfectly equivalent when the spillover parameters in both models are set equal to zero. One can go back and forth from one model to the next simply by applying the inverse mapping (where  $x_i$  is firm  $i$ ’s cost reduction)

$$x_i = \sqrt{\frac{1}{\gamma} y_i} \text{ if and only if } y_i = \gamma x_i^2$$

<sup>5</sup>Indeed, a short computation shows that the inequality  $\frac{(2-\beta)a}{9c} < \frac{(1+\beta)(2-\beta)}{27\beta}$ , which is part of (10), holds if and only if  $\frac{a}{c} < \frac{(1+\beta)}{3\beta}$ . Since  $\frac{a}{c} > 2$  by Assumption (A1), the latter inequality requires that  $\beta < 0.2$ .

any level of R&D costs satisfying our basic assumptions), but face the right incentives for private R&D. At the other extreme of full spillovers ( $\beta = 1$ ), it is well known that the incentives for R&D are strongly diluted by the free rider problem, but firms are free of the prisoner's dilemma. It is thus of interest to compare firms' equilibrium profit at these two extreme levels of spillovers to determine which of the two impediments is more detrimental to profits, the prisoner's dilemma or the lack of appropriability of know-how. Denote by  $\Pi^*(\beta)$  the per-firm equilibrium profit for the non-cooperative R&D scenario.

**Proposition 2** *For the interior equilibrium, per-firm equilibrium profit  $\Pi^*(\beta)$  satisfies:*

- (i)  $\Pi^*(\beta)$  is higher under full spillover ( $\beta = 1$ ) than under no spillover ( $\beta = 0$ ).
- (ii)  $\Pi^*(\beta)$  increases in  $\beta$  on  $[0, \beta^*)$ , reaches its maximum at  $\beta^*$ , and decreases in  $\beta$  on  $(\beta^*, 1]$ , where  $\beta^*$  depends on the model parameters and is always in the interval  $(0.59, 0.73)$ .

**Proof.** (i) For  $\beta = 0$  and  $\beta = 1$ , the per-firm equilibrium profits are, respectively,

$$\begin{aligned}\Pi_{\beta=0}^* &= \frac{\gamma(9b\gamma - 4)(a - c)^2}{(9b\gamma - 2)^2} \\ \Pi_{\beta=1}^* &= \frac{\gamma(18b\gamma - 1)(a - c)^2}{2(9b\gamma - 1)^2}.\end{aligned}$$

To compare the above profits we look at the difference  $\Delta\Pi^* = \Pi_{\beta=1}^* - \Pi_{\beta=0}^* = \frac{\gamma(4+27b\gamma(9b\gamma-2))(a-c)^2}{2(9b\gamma-2)^2(9b\gamma-1)^2}$ , which is positive for any  $b\gamma > \frac{2}{9}$ . Therefore  $\Pi_{\beta=1}^* > \Pi_{\beta=0}^*$ .

(ii) Differentiating per-firm equilibrium profit,

$$\Pi^* = \frac{\gamma(9b\gamma(1 + \beta) - (2 - \beta)^2)(a - c)^2}{(9b\gamma - 2 + \beta)^2(1 + \beta)},$$

with respect to  $\beta$  to find

$$\frac{\partial\Pi^*}{\partial\beta} = \frac{\gamma(27b\gamma(2 - \beta(2 + \beta)) + (-2 + \beta)^3)(a - c)^2}{(9b\gamma - 2 + \beta)^3(1 + \beta)^2},$$

where the denominator is positive per our assumptions. Then the conclusion follows from studying the sign of the cubic expression

$$\text{sign}\left[\frac{\partial\Pi^*}{\partial\beta}\right] = \text{sign}[27b\gamma(2 - \beta(2 + \beta)) + (-2 + \beta)^3]$$

The computational details are left out. ■

Part (i) of this result is somewhat surprising and provides some support for the importance of understanding the prisoner's dilemma in R&D. This result confirms the potency of the prisoner's dilemma in the quantitative sense that it appears to be more detrimental to profit than the well known free rider problem. Part (ii) further elaborates on the importance of the prisoner's dilemma in R&D since firms' profits are maximized by a relatively high value of the spillover parameter lying well beyond the range validating a prisoner's dilemma in R&D (see Stepanova, 2009).

While the fact that equilibrium profit is relatively high at full spillovers might appear surprising, it may simply be a reflection that equilibrium R&D levels tend to be excessive in such two-stage games because firms are able to foresee that a high level of R&D leads to a higher market share at the Cournot stage (Brander and Spencer, 1983). Thus, by diluting firms' incentives for R&D, a high level of spillovers helps firms mitigate the tendency for excessive R&D. This also squares well with the fact that a prisoner's dilemma in R&D does not prevail at high levels of spillover effects.

### 3.4 Welfare comparison

In this subsection, the focus is on the second-best social planner view of the comparison between the noncooperative R&D equilibrium  $(y^*, y^*)$  and the no-R&D outcome. Adopting standard usage, the qualifier "second-best" applied to the social planner's conduct captures the common situation (used as a useful benchmark) in which the planner is empowered to decide on the two firms' R&D levels but not on the firms' market conduct. It is then natural to postulate that the firms will continue to be Cournot competitors in the product market, as opposed to say perfect competitors (as would be the case under first-best planning).

The usual Marshallian social welfare for the second-best benchmark is given by (given equilibrium R&D level  $y^*$  by each firm)

$$W(y^*, y^*) = \int_0^{2q^*} (a - bt)dt - 2(c - \sqrt{(1 + \beta)y^*/\gamma})q^* - 2y^*,$$

where

$$q^* = \frac{1}{3b}[a - c + \sqrt{(1 + \beta)y^*/\gamma}]$$

is per-firm equilibrium Cournot output when both firms 1 and 2 have unit costs  $c - \sqrt{\frac{1}{\gamma}(1 + \beta)y^*}$ .

Upon some computation, the social welfare level when firms choose R&D levels  $(y^*, y^*)$  is given by

$$W(y^*, y^*) = \begin{cases} \frac{2\gamma(a-c)^2 [18\gamma b(\beta+1) - (\beta-2)^2]}{(9b\gamma + \beta - 2)^2 (1 + \beta)} & \text{if } b\gamma > \frac{(2-\beta)a}{9c} \\ \frac{2(2a^2(\beta+1) - 9bc^2\gamma)}{9b(\beta+1)} & \text{if } b\gamma \leq \frac{(2-\beta)a}{9c} \end{cases} \quad (12)$$

In line with earlier practice, the top and bottom lines in (12) give the equilibrium welfare level when the (common) Nash equilibrium R&D level is interior and on the boundary, respectively.

**Proposition 3** *Conducting the Nash equilibrium levels of R&D always leads to a higher social welfare than no R&D, or  $W(y^*, y^*) > W(0, 0)$ .*

**Proof.** From standard Cournot theory, we know that in the case of no R&D, or R&D decisions  $(0, 0)$ , the welfare level is given by  $W(0, 0) = \frac{4(a-c)^2}{9b}$ . Comparing this with  $W(y^*, y^*)$ , simple computations lead to the desired result. ■

Thus the prisoner's dilemma always works to the advantage of society. In other words, the resulting R&D, which is something firms wish they could avoid whenever spillover effects are small, improves consumer surplus by leading to a higher industry output. Furthermore, this positive effect on consumer surplus clearly outweighs the negative effect on producer surplus (reflected in the prisoner's dilemma), since overall social welfare improves.

While the result that R&D can constitute bad news for competing firms is somewhat counter-intuitive, the fact that R&D is advantageous to society as a whole is fully in line with standard conventional wisdom on the overall cost and benefit effects of R&D. Indeed, it is often argued that the social returns to R&D far exceed the private returns. This is widely confirmed by the extensive empirical literature on the economics of innovation, see e.g., Bernstein and Nadiri (1988), Griliches (1995), Hall (1996) and Scotchmer (2006).

## 4 Comparison with the AJ model: Spillovers in R&D outputs

This section compares the results of the present paper to similar findings obtained in the context of the other model of non-cooperative R&D with spillovers, firstly introduced by d'Aspremont and

Jacquemin (1988), henceforth AJ. In order to make this paper self-contained, we provide a brief summary of the AJ model.

It is also a two-stage game of R&D/product market competition that shares everything with the KMZ model except the following important feature. In stage one, firms simultaneously decide on cost reducing levels  $x_1$  and  $x_2$  (instead of R&D expenditures) and the spillover effects operate directly on cost reductions. In other words, the post R&D unit cost of firms 1 and 2 are given respectively by

$$c - x_1 - \beta x_2 \text{ and } c - x_2 - \beta x_1$$

where  $\beta \in [0, 1]$  is the spillover parameter. The cost corresponding to an autonomous cost reduction  $x_i$  is  $\gamma x_i^2$ , where  $\gamma > 0$ , indicating decreasing returns to R&D.

Given the unique Cournot equilibrium in the second stage, the payoff of firm  $i$  as a function of the two R&D decisions is given by

$$\begin{aligned} \pi_i(x_i, x_j) &= \frac{1}{9b} [a - 2(c - x_i - \beta x_j) + (c - x_j - \beta x_i)]^2 - \gamma x_i^2 \\ &= \frac{1}{9b} [a - c + (2 - \beta)x_i - (1 - 2\beta)x_j]^2 - \gamma x_i^2 \end{aligned}$$

Bacchiega et al. (2010) derive the result that the three stage version of the AJ model, with stage one corresponding to a binary decision to perform R&D or not, leads to a prisoner's dilemma in R&D if and only if the spillover parameter is small enough (specifically,  $\beta < 0.2$ ). Amir et al. (2011) derive a similar result but using directly the two-stage AJ model, upon proposing a meaningful definition of a prisoner's dilemma for games with a continuum action set.

The present paper may thus be seen as a robustness check on these two papers, based on using the more generically valid model of non-cooperative R&D with spillovers, the KMZ model, as argued by Amir (2000).<sup>6</sup> In light of all the important divergences between the two models uncovered and discussed in detail in Amir (2000), it certainly cannot be assumed on a priori grounds that the two models would be in agreement as to the presence of a prisoner's dilemma in R&D. Nevertheless, the results on this important issue at hand turn out ex post to be in agreement in a qualitative sense between the two models. Indeed, both feature a prisoner's dilemma in R&D for sufficiently small values of the spillover parameter. In particular, it is fully expected that the two models would agree

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<sup>6</sup>Other studies have subsequently elaborated on this point, see, e.g., Martin (2002) and Hauenschild (2003).

in the absence of spillovers (i.e., for  $\beta = 0$ ). On the other hand, there is a difference of a quantitative nature between the results of the models. The scope for a prisoner’s dilemma (as measured by the size of the sub-region in  $(\beta, \gamma)$  space where it holds) is indeed a bit broader in the AJ model, as can be seen from the graphs depicted in Figure 1. This is to be expected since the equilibrium outcomes of the two models would coincide if the R&D cost function for the AJ model were changed to

$$y_i = \gamma(1 + \beta)x_i^2$$

i.e., if the cost function were shifted upwards to undo the excessive propensity towards high R&D levels generated by that model, as argued in Amir (2000). This excessive R&D propensity is due to the fact that spillover effects are additive in cost reductions, or R&D outputs, thus in a way getting around the important property of decreasing returns to scale of the R&D process.

In conclusion, the qualitative agreement between these two different models on the presence of a prisoner’s dilemma for small values of the spillover parameter reinforces the validity of the result and provides a useful robustness check. It is also worth noting that the range of spillovers for which the result holds covers what one might expect to be much of the relevant range for real-life spillovers in various industries.

## 5 Cooperative versus non-cooperative R&D

In this section, we discuss how the presence of a prisoner’s dilemma might shed light on the comparison between non-cooperative R&D and cooperative R&D, in terms of the incentives faced by R&D-cooperating firms relative to full non-cooperation. The results of this subsection are of independent interest for the literature on research joint ventures (of which Katz, 1986, AJ and KMZ are some of the most prominent papers, also see Ruff, 1969), since they address some novel questions of interest vis a vis that literature.

A recent strand of the literature on innovation in industrial organization has investigated the merits of R&D cooperation among firms that remain competitors in product markets (see Katz, 1986, AJ, KMZ, Amir et al., 2003, among many others). One frequent comparison in this literature is between the scenario of non-cooperative R&D, as described in Section 1, and one particular scenario of R&D cooperation, called an R&D cartel, wherein firms jointly choose their levels of



R&D expenditures in the first stage with a view to maximize their joint overall profits in the two-stage game, knowing that they will move to the second stage of the game where they will continue to be Cournot competitors. In other words, in terms of the notation from Section 2, the joint objective of the R&D cartel is then

$$\max_{y_1, y_2} \{\Pi_1(y_1, y_2) + \Pi_2(y_1, y_2)\}$$

Assuming a symmetric solution<sup>7</sup>, the equilibrium R&D level for this scenario is easily computed to be (see e.g., KMZ or Amir, 2000)

$$y^C = \begin{cases} \frac{(1+\beta)(a-c)^2\gamma}{(9b\gamma-1-\beta)^2} & \text{if } b\gamma > \frac{(2-\beta)a}{9c} \\ \frac{\gamma c^2}{(1+\beta)} & \text{if } b\gamma \leq \frac{(2-\beta)a}{9c} \end{cases}$$

For simplicity, in what follows, we concentrate our comparisons to the interior equilibria for both solutions.<sup>8</sup>

The cartel equilibrium profit per firm is

$$\Pi^C = \frac{(a-c)^2\gamma}{9b\gamma - (\beta+1)}$$

The first result below expresses properties of the effective cost reductions, defined for the non-cooperative and cooperative cases by, respectively (the explicit dependence on  $\beta$  will be useful notation below)

$$X^*(\beta) = \sqrt{\frac{1}{\gamma}(1+\beta)y^*} \text{ and } X^C(\beta) = \sqrt{\frac{1}{\gamma}(1+\beta)y^C}$$

Since these quantities take into account the effects of own R&D as well as rival's R&D on one's production costs, they constitute the right measure of the speed of technological progress (or actual cost reduction) in an industry.

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<sup>7</sup>This assumption is not fully justified a priori, as analyzed in detail by Salant and Shaffer (1998, 1999), who show that in some cases, the optimal global solution may be asymmetric. However, we follow much of the literature in ignoring this subtlety.

<sup>8</sup>Observe that the cartel and non-cooperative solutions are not interior for the same parameter values. Thus, if the spillover rate is small, i.e.,  $\beta < 1/2$ , then  $\frac{a(2-\beta)}{c} < \frac{a(1+\beta)}{c}$ . If there is a range of R&D effectiveness levels  $\gamma$  such that  $\frac{a(1+\beta)}{c} < b\gamma < \frac{a(2-\beta)}{c}$ , then cooperative firms choose interior equilibrium levels, while in case of noncooperation the firms choose maximal possible investment (the boundary equilibrium). Likewise, in case of large spillover, the opposite inequality holds  $\frac{a(2-\beta)}{c} < b\gamma < \frac{a(1+\beta)}{c}$ , hence for these R&D effectiveness levels, noncooperative firms choose an interior level of R&D investments, while cooperative firms choose the maximal one (boundary solution).

Upon a short computation, one finds that the interior equilibrium levels are (see e.g., Amir, 2000):

$$X^*(\beta) = \frac{(2 - \beta)(a - c)}{9b\gamma - 2 + \beta} \quad (13)$$

and

$$X^C(\beta) = \frac{(1 + \beta)(a - c)}{9b\gamma - 1 - \beta} \quad (14)$$

The first observation deals with the comparative statics of R&D with respect to the spillover parameter. The finding is that, while the equilibrium R&D levels of both scenarios respond monotonically to changes in  $\beta$ , they do in opposite directions in a global sense.

**Lemma 4** *The following comparative statics for the equilibrium R&D levels hold;*

- (i)  $X^*(\beta)$  is decreasing in  $\beta \in [0, 1]$ . Hence  $\max_{\beta} X^* = X^*(0)$  and  $\min_{\beta} X^* = X^*(1)$ .
- (ii)  $X^C(\beta)$  is increasing in  $\beta \in [0, 1]$ . Hence  $\max_{\beta} X^C = X^C(1)$  and  $\min_{\beta} X^C = X^C(0)$ .
- (iii)  $X^*(0) = X^C(1)$  and  $X^*(1) = X^C(0)$ .

**Proof.** (i)-(ii) Using the expressions in (13)-(14), a simple computation establishes that  $dX^*(\beta)/d\beta < 0$  and  $dX^C(\beta)/d\beta > 0$  (under our parameter restrictions). The other statements in (i)-(ii) then follow directly as consequences of these two properties.

(iii) This follows directly from (13)-(14). ■

Thus as the spillover rate  $\beta$  increases, cartel R&D increases, while non-cooperative R&D decreases. In addition, an interesting type of symmetry holds: the end level of one curve (at one extreme of the value of  $\beta$ ) is the starting level of the other (at the other extreme of the value of  $\beta$ ). Neither of these statements is surprising since it is well known that for  $\beta = 1/2$ , the two levels of R&D are equal (see e.g., KMZ or Amir, 2000). This result is also quite intuitive. For the non-cooperative case, the result is widely known, and a simple consequence of the standard free-riding effect in the commons, which should clearly get worse as  $\beta$  increases. For the cartel, due to the internalization of the R&D externality, a higher value of  $\beta$  simply translates into a more effective joint R&D process for the cartel, which therefore leads to higher equilibrium R&D levels.

In light of these simple comparative statics properties of the effective cost reductions, the presence of a prisoner's dilemma for small values of the spillover parameter is not surprising. Indeed, in

a neighborhood of  $\beta = 0$ , the noncooperative solution leads to the highest possible level of R&D<sup>9</sup> while the R&D cartel calls for the smallest level of R&D for that scenario. Since the cartel solution allows firms to internalize the underlying externalities, it can be expected from this proposition that a prisoner's dilemma in R&D prevails for small values of the spillover parameter.

Conversely, in a neighborhood of  $\beta = 1$ , the cartel solution calls for its maximal level of R&D while the noncooperative solution leads to its minimal level of R&D. From these observations, one can infer that there is no prisoner's dilemma at work in this case.

In order to shed further light on the incentive for R&D cooperation, we now consider the difference in per-firm profit  $\Pi^C - \Pi^*$  as a natural measure of the incentive for R&D cooperation, and investigate how it changes with the spillover parameter  $\beta$ . The result is provided next.

**Proposition 5** *As  $\beta$  increases from 0 to 1, we have:*

- (i)  $\Pi^C - \Pi^* \geq 0$  with equality if and only  $\beta = 1/2$ .
- (ii)  $\Pi^C - \Pi^*$  decreases in  $\beta$  for  $\beta \in [0, 1/2]$ , reaches its minimum of zero for  $\beta = 1/2$ , and then increases in  $\beta$  for  $\beta \in [1/2, 1]$ .
- (iii)  $\Pi^C - \Pi^*$  is higher at  $\beta = 0$  than at  $\beta = 1$ , assuming that  $9b\gamma > 2$ .

**Proof.** (i) Since the cartel can always elect the non-cooperative levels of R&D as one of its possible choices, it follows immediately that  $\Pi^C - \Pi^* \geq 0$ . (Note that this is a well known and very general result).

The fact that the two profit levels are equal for  $\beta = 1/2$  follows directly from the well-known fact that the R&D levels are equal for the two scenarios if and only if  $\beta = 1/2$  (see KMZ or Amir, 2000).

(ii) A long but simple computation shows that  $\frac{\partial}{\partial \beta} (\Pi^C - \Pi^*)$  is equal to a positive number (depending in a complex way on all the parameters of the model) multiplied by  $(2\beta - 1)$ . The conclusion follows then directly from this computation, combined with part (i).

(iii) This follows from a tedious but straightforward computation comparing  $\Pi^C(0) - \Pi^*(0)$  and  $\Pi^C(1) - \Pi^*(1)$  (note that  $9b\gamma > 2$  is simply the second order condition when  $\beta = 0$ ). The computational details are left out. ■

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<sup>9</sup>It is worthwhile to recall that  $\max_{\beta} X^* = X^*(0)$  is also equal to  $\max_{\beta} X^C = X^C(1)$ , which is by definition the equilibrium R&D level of the cartelized research joint venture (as in KMZ) or the joint lab (Amir, 2000).

It is generally believed that the incentive for R&D cooperation should be higher in industries characterized by substantial spillover effects, and more precisely that this incentive ought to be increasing in the spillover parameter. Indeed, the less perfectly appropriable R&D is, the more free-riding is expected to take place, and the less efficient the non-cooperative scenario is expected to perform. In this vision, by internalizing the public goods externality of R&D, the cartel would be expected to provide a correction to this problem, whose benefits would be higher the more substantial the spillover rate is. The above result shows that this arguably common intuition is actually valid only for relatively high spillover levels, those exceeding  $1/2$ . For such spillovers, forming an R&D cartel is indeed a way to circumvent the lack of incentive for private R&D, as is commonly understood.

However, the incentive for R&D cooperation is actually very high for very small spillover values and in fact maximal for zero spillovers (see part (iii) of the Proposition). The fact that this incentive is decreasing in  $\beta$  for  $\beta \in [0, 1/2]$  is clearly not compatible with the aforementioned commonly held intuition, and thus emerges as a rather surprising finding here.

The presence of a prisoner's dilemma for the non-cooperative solution can provide an insightful lens through which to take a fresh look at this finding. Indeed, R&D cooperation in the form of an R&D cartel is a very natural way for firms to escape from the destructive grip of a prisoner's dilemma. First observe that the latter prevails only for small values of the spillover parameter, i.e.,  $\beta \in [0, 1/5]$ , and its scope can be seen as decreasing in  $\beta$  over this range, in the sense that it holds for a smaller range of values of  $\gamma$  (a measure of the cost of R&D). Thus, as  $\beta$  increases in the range  $[0, 1/5]$ , the prisoner's dilemma gets more and more attenuated and the incentive for R&D cooperation decreases. In this perspective, R&D cooperation can be seen as motivated by a desire on the part of firms to avoid the grip of a prisoner's dilemma in their R&D decisions for small spillovers.

On the other hand, for high spillovers, R&D cooperation should be seen as the proper response to a need to internalize the public goods externality of R&D, as is widely believed (see e.g., AJ or Spence, 1984). In light of these two complementary explanations, it should then not come as a surprise that, for intermediate values of spillovers (those close to  $1/2$ ), the incentive for R&D cooperation is indeed minimal (in fact, non-existent for  $\beta = 1/2$ ). In conclusion then, the mutu-

ally exclusive nature of the prisoner's dilemma and significant spillover levels offers a novel and interesting perspective to explain the incentives to form R&D cartels.

We end with a final remark on the antitrust implications of the results of this section. Since R&D cooperation allows firms to avoid a prisoner's dilemma in R&D in industries with small spillover levels and since this prisoner's dilemma is actually favorable to consumers and society, the usual permissive attitude of antitrust authorities concerning R&D cooperation among firms that continue to compete in product markets might require some revision. This permissive attitude is fully justified for industries with high levels of spillovers since it allows firms to circumvent the effects of the appropriability externality and induces them to increase their R&D levels relative to the noncooperative R&D scenario. However, for industries with low levels of spillovers, it would be more appropriate to limit firms' cooperation in R&D, thus forcing them to face the socially beneficial prisoner's dilemma at hand.

## 6 Conclusion

This paper has confirmed the presence of a standard prisoner's dilemma in R&D decisions for (a modified version of) one of the commonly used models of strategic R&D and product market competition, the so-called KMZ model, in which spillovers are postulated to take place in R&D inputs. The characterization of the parameter region under which this dilemma arises, low R&D costs and low spillovers, is very similar to that for the other model, due to AJ, which postulates spillovers in R&D outputs instead. This result is not easily anticipated on intuitive grounds since the two models are known to have substantial differences on some key issues (Amir, 2000).

In terms of consumer surplus and social welfare, this prisoner's dilemma always works in favor of consumers and society as a whole, just as one would expect from an intuitive standpoint.

The presence of this prisoner's dilemma can be invoked to shed light on a number of different findings in the broader R&D literature. First we show that equilibrium profits in the R&D game are higher at full spillovers than at zero spillovers, suggesting that the prisoner's dilemma is more detrimental to firm profits than the public goods nature of know-how. A second finding concerns the incentives firms face for forming R&D cartels. Such cartels are usually understood as appropriate in cases where R&D spillovers are substantial, as a way to internalize the common property externality

of knowledge. It turns out that the incentive to form an R&D cartel is actually maximal in the case of zero spillovers. Since the scope of the prisoner's dilemma increases as the spillovers go down, one can argue that R&D cartels are motivated on the part of the firms involved by the search for a way out of the prisoner's dilemma grip, whenever spillover effects are relatively small. When the latter are large, the commonly held view that they can motivate firms to form R&D cartels (forcefully put forth by AJ and others) is implicitly confirmed here.

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